A Model for Worker Honeybees
Building the Triggers of Honeycomb Construction Process

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Abstract: Honeycombs consist of a regular configuration of hexagonal holes. Although this self-organized and precise architecture has attracted much research, the rules by which it is constructed have not yet been elucidated. Here, we propose a simple and instinctive model for worker honeybees building a honeycomb. We assume that the wax cluster is dug linearly, guided by the workers’ sense of touch. The regularity is not then produced intentionally; it is due to the competition between the growth of the wax and its excavation by the bees. We carried out two-dimensional simulations of our model, and obtained triangular and fishbone structures that are the triggers of the honeycomb construction process. Our study clarifies that anisotropic growth of the wax is a key factor in the construction.

Keywords: honeybee, honeycomb construction, self-organizing process, Eden growth, agent simulation.

1. INTRODUCTION

Honeybees are a leading example of social insects; they live communally and care cooperatively for their young [1]. Many social insects build their own nests, and worker bees construct their nest without any supervision, i.e., honeycombs are constructed in a self-organizing process. The precise nature of this structure has long attracted both biologists and engineers; however, the mechanism of the construction is still an open problem. Honeycombs have a plate-like structure with regularly spaced hexagonal holes in both sides. They are made of beeswax, which is secreted by the worker honeybees, and they are constructed in confined spaces, such as hollows and beehives. In the construction of honeycombs, wax is first attached to the ceiling of the space, and this is then built up. It grows into a triangular pyramid, and then it grows horizontally to produce fishbone structures (Fig. 1). A three-dimensional structure is achieved by wax being attached successively in the vertical direction.

There is an ongoing debate about how bees produce this precise hexagonal configuration. There are two general standpoints: one is that honeybees are exquisite engineers, and the other is that the precise structure is simply a result of the laws of physics. Recently, Karihaloo et al. [2] reported that, due to surface tension, the softened wax is spontaneously thinned and becomes hexagonal. There are several varieties of bees that construct their nests from woody fibers, and in these nests, the frames of the holes are not hexagonal. Therefore, we are interested in how bees build a regular pattern of holes.

We propose a simple phenomenological model for worker honeybees building a honeycomb [3]. As mentioned above, there is no leader required to build it. In addition, honeybees do not possess big brains, and their lifespans are not sufficiently long to learn how to make such a precise and regular structure. Thus, the construction rule should be simple and instinctive. Our most fundamental hypothesis is that there are two types of worker honeybees: suppliers, who create the wax, and excavators, who shape it. Although in nature, all workers play both roles, in the model, their functions are clearly divided. We assume that the excavators dig wax in a linear fashion, and this implies that they do not intend to dig regularly arranged holes. We also assume that the excavators’ sense of touch prevents them from penetrating the wax clusters, and this assumption plays an important role in our model.

2. MODEL

We consider a two-dimensional lattice in which the grid interval is $\Delta r$, and each cell can contain a unit of wax. In the model, the worker honeybees are categorized into two types: the suppliers and the excavators. We will describe these roles in the following subsection.

2.1. The Growth of the Comb

In each unit time step, beeswax is added to preexisting wax. We assume that the suppliers attach wax in an anisotropic way: the suppliers recognize a certain direction (here, the $x$-direction), and they prefer to add wax in

\[\text{Fig. 1 A fishbone structure (white) is observed at the root of a honeycomb attached to the ceiling of a space. The red square in the figure emphasizes a triangular pyramid. The fishbone structure consists of a sequence of triangular pyramids.}\]
Fig. 2 An excavation area (gray) consists of a box with a semicircle in front. The speed is $v_e$, the height is $h_e$, and the width is $w_e$. The broken line indicates the maximum distance $d_1$ within which the excavator can use the sense of touch to detect wax or other excavators. The black point indicates the center of rotation of the excavation area.

Fig. 3 Snapshot of a simulation. For the same data, the left panel shows a wax cluster (white) and the excavation areas (yellow), while the right panel shows only the wax cluster. The areas (green) around the excavation areas represent the area within $d_1$, in which the excavator can detect wax and other excavation areas. Note that the fishbone structure is developing.

The dynamics of the suppliers is not tracked. Instead, the wax grows through an anisotropic Eden growth process. In the original Eden process [4], the growth of clusters, such as material deposits and bacterial colonies, is due to random accumulations on their boundaries. In contrast, the anisotropic Eden model produces directionally weighted accumulations.

The suppliers produce wax without limit, although without the excavators, it does not result in honeycomb.

2.2. Excavation

We consider the excavation area in which the excavators dig into the wax cluster. The excavators are modeled as two connected segments: a head and a body. The head segment can rotate to plus or minus $\pi/2$, and the connection point (i.e., neck) is the center of rotation. The body of the excavator moves forward and digs into the wax cluster. These assumptions result in a characteristic form for the excavation area: a box with a semicircle in front (Fig. 2). We assume that all excavation areas have the same form.

The sense of touch of the excavator plays two important roles in the model. First, within the range of $d_1$, the excavator can recognize wax and excavation areas. Thus, when an excavator senses a wax cluster in front of it, it rotates so that it approaches it, but when an excavator encounters another excavation area, it jumps to a random point. In addition, an excavator can measure the depth of a wax cluster. If the depth in front of the excavation area is less than $d_2$, the excavator does not move forward, but instead, it rotates through a small angle and searches for a thicker location.

The excavation area continuously moves within the system. Furthermore, the excavators move continuously, except when encountering another excavation area, encountering a wax cluster, or detecting that the depth of the wax cluster is less than $d_2$. The speed of all excavation areas is set to be $v_e$.

3. RESULTS AND DISCUSSION

At the onset of the simulation, only the center cell contains a unit of wax, and the excavation areas are randomly distributed in the system. The details of the simulation will be discussed in the presentation.

The wax cluster grows over time, and the excavators shape it by digging. The triangular pattern was successfully built, and it forms the base of the three-dimensional triangular pyramid structure. Due to the anisotropic wax growth in the $x$-direction, a fishbone-like pattern was obtained, as shown in Fig. 3. However, the pattern is not very regular. Thus, further adjustment to the model is needed; this may include the size of excavation area, the velocity of the excavation, and the rate at which wax is created.

If the growth is assumed to be isotropic, the wax clusters are almost disk-like. Anisotropic growth is therefore a necessary condition for obtaining a fishbone structure. It is important to investigate how the anisotropy is produced.

In conclusion, we have proposed a model in which worker honeybees build triangular and fishbone patterns, which are the triggers of the construction of honeycomb. Our study reveals that anisotropic growth is a key to understanding the rules for constructing honeycomb. It is known that the honeycomb structure is strong, reduces the cost of materials, and has a high storage capacity. Since the model can be applied to a three-dimensional system, our study might enable the construction of nanoscale honeycomb structures.

REFERENCES