A Study on Rate-Dependent Hysteresis Compensation of Piezo-Ceramic Actuator

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Abstract: This paper concerns the tracking control problem of a piezo-ceramic actuator with hysteresis. Several works have been carried out to suppress the hysteretic motion using the numerical inverse models. However, it is known that hysteresis properties of piezo-ceramic actuators crucially depend on the frequency of input signals, which is referred to as the rate-dependent hysteresis. This paper handles the compensation of rate-dependent hysteresis of a piezo-ceramic actuator. The enhanced Play and Stop models capable of representing the inverse hysteretic mapping for frequencies where no identification has been performed will be proposed and incorporated into the feed-forward control system to compensate the rate-dependent hysteresis. Experimental results show that proposed controllers can maintain Root-Mean-Square error lower than 0.5 μm for wide range of different input frequency.

Key-Words: *Piezo-ceramic actuator, hysteresis, Play model, Stop model, rate-dependent hysteresis.*

1. Introduction

Piezo-ceramic actuators have advantages in high precision positioning, high force output and fast transient responses. They are thus commonly used in areas which require precise micro-positioning control such as micro stages and micro assembly systems. However, piezo-ceramic actuators exhibit serious hysteresis properties, and performance of the tracking control deteriorates accordingly. Numbers of studies have been conducted to overcome the performance degradation caused by hysteresis phenomena and many different models capable of representing the input-output hysteretic characteristics have been proposed [1]-[4].

Compensation of hysteresis has also been studied intensively using the models developed in the literature. The main idea of hysteresis compensation based on these models is to form the numerical inverse hysteresis mapping and use it as a feed-forward controller[3, 4]. It is a common practice to combine feedback controllers with those feed-forward controllers to gain some performance robustness. However, these hysteresis models require identification procedure which is dependent on the frequency of the input signal, and the feed-forward controllers become less effective if the frequency of the reference signal is not equal to the one used in the identification process.

This paper treats the compensation of frequency dependent hysteresis which is hereafter called the rate-dependent hysteresis in micro positioning control. Recently, Xu and Li[5] proposed a novel model to describe rate-dependent hysteresis and used it for compensation. The advantage of the model is that it uses only 9 parameters to be determined. However it may encounter difficulty in treating compensation of minor loop hysteresis. This study involves in designing a feed-forward controller for rate-dependent hysteresis which is based on the Play or the Stop models. Experimental results show the effectiveness of the proposed feed-forward controller for compensating both major and minor hysteresis loops.

Received: 21 April 2012, Accepted: 23 May 2012
Available online 17 July 2012
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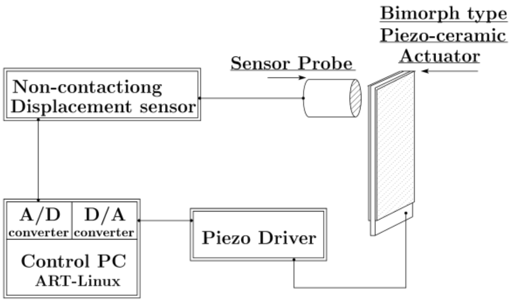

Figure 1 Schematic description of the experimental apparatus

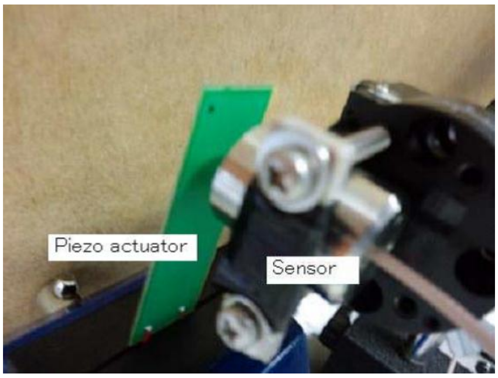

Figure 2 Piezo actuator and the sensor probe

2. Experimental system

The piezo-ceramic actuator studied in this paper is a bimorph type actuator, which has an effective length of 48 mm and a displacement magnitude of 0.6 mm / at 70 V. Fig.1 shows the setup of the experimental system. The system is controlled in real time by a PC in which ART-Linux[6] is installed as a real time operating system. A non-contacting displacement sensor is used to measure the tip displacement of the piezo-ceramic actuator. The sensor used is model M-2213 manufactured by MESS-TEK corporation. It has dynamic range of ± 1000 μm and a resolution of 20 nm. Fig.2 shows the photo of the piezo-ceramic actuator and the sensor. Hysteresis of a piezo-ceramic actuator changes its characteristics as the function of reference input frequency. This kind of hysteresis is called the rate-dependent hysteresis[5]. The shape of input/output trajectory changes drastically as the frequency of the reference sinusoidal input increases. The piezo-ceramic actuator used in this research also exhibits rate-dependent hysteresis characteristics as shown in Fig.3.

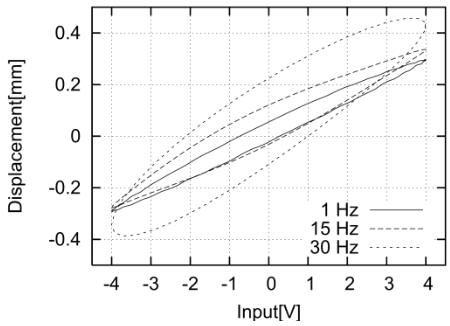

Figure 3 An example of rate-dependent hysteresis observed in our experiments

3. Hysteresis modelling

Several models have been proposed to describe the hysteresis characteristics mathematically. They are used to form a feed-forward compensator to cancel out the hysteresis for control purpose. This paper proposes new modelling method capable of describing inverse characteristics of the rate-dependent hysteresis of a piezo-ceramic actuator, which is called the enhanced Play / Stop models. The enhanced Play / Stop models owe their basis to the distribution function of the Preisach model. The outline of these three models will be given here prior to describing our development on modelling inverse characteristics of the rate-dependent hysteresis.

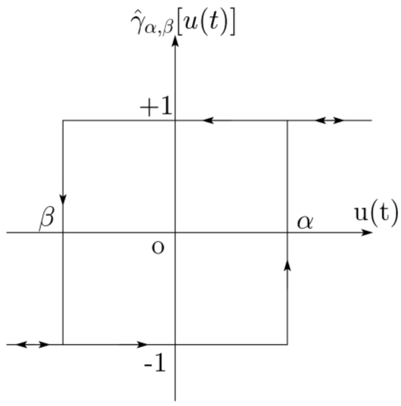

Figure 4 Hysteresis operator $\hat{\gamma}_{\alpha,\beta}[u(t)]$

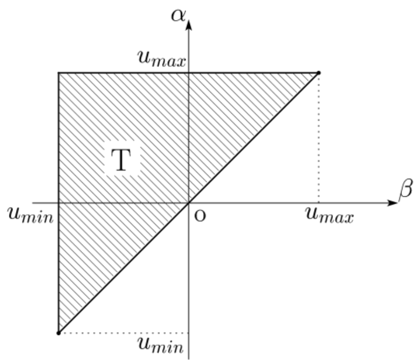

Figure 5 The Preisach plane *T*

3.1 Preisach model

The Preisach model can be written mathematically as

$$\mathrm{F(u)} = \iint_T K(\alpha,\beta)\hat{\gamma}_{\alpha,\beta}[u(t)]d\alpha d\beta \qquad (1)$$

where *F*(*u*) is the system output, *K*(*α, β*) is a distribution function and $\hat{\gamma}_{\alpha,\beta}[u(t)]$ is the hysteresis operator having an output of +1 or -1 with *α* and *β* corresponding to up and down switching values of the operator as shown in Fig. 4, respectively. The distribution function *K*(*α, β*) determines the shape of the hysteresis curve and can be identified by experiments. The domain of integration *T* in equation (1) which is called the Preisach plane is depicted in Fig.5.

The Preisach model (1) is used in the discretized form in the practical implementation. The Preisach plane and the corresponding distribution function should be discretized accordingly. Let the interval $[u_{min}, u_{max}]$ be discretized into *N* small intervals $[\mathrm{u_{n-1}}, \mathrm{u_n}]$ $(\mathrm{n} = 1,2,\cdots,\mathrm{N})$. The discretized distribution function $\widehat{\mathrm{K}}(\mathrm{i,j})$ is defined accordingly by

$$\widehat{K}(\mathrm{i,j}) = \int_{u_{j-1}}^{u_j}\int_{u_{i-1}}^{u_i} K(\alpha,\beta)d\alpha d\beta, \qquad (2)$$

which corresponds to (*i*, *j*)-th cell of a discretized Preisach plane. The discrete version of equation (1) is given by

$$\mathrm{F(u)} = 2\sum_{i=1}^{N}\sum_{j=1}^{i}\widehat{K}(i,j)\widehat{T}(i,j) - \sum_{i=1}^{N}\sum_{j=1}^{i}\widehat{K}(i,j), \qquad (3)$$

where $\widehat{\mathrm{T}}(\mathrm{i,j})$ represents how (*i*, *j*)-th cell is magnetized. $\widehat{\mathrm{T}}(\mathrm{i,j})$ takes the value 1 when the cell is positively magnetized, otherwise it takes the value 0.

3.2 Play model

The Play model describes the hysteretic relation between the input *u*(*t*) and the output *F*(*u*) with the equation defined by

$$\mathrm{F(u)} = \sum_{n=1}^{N_p} f_n\left(p_n\big(u(t)\big)\right), \qquad (4)$$

where $\mathrm{N_p}$ is the number of Play hysteron operators, $\mathrm{p_n}$ is the *n*-th Play hysteron operator, and $\mathrm{f_n}$ is a single-valued shape function corresponding to $\mathrm{p_n}$. The Play model is schematically depicted in Fig.6. The Play hysteron operator $\mathrm{p_n}$ is defined by

$$p_n\big(u(t)\big) = \max(\min(p_n^0, u(t) + \zeta_n), u(t) - \zeta_n) \qquad (5)$$

where p_n^0 is the value of $\mathrm{p_n}$ at the previous sampling instance and ζ_n is a constant representing the width of the artificially introduced dead zone. ζ_0 should be chosen as 0 to represent memory-less part of the response. The behaviour of a Play hysteron operator p_n with saturation is illustrated in Fig.7.

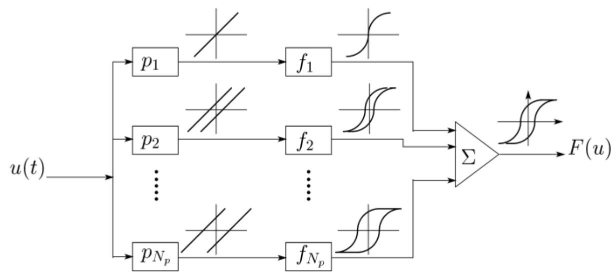

Figure 6 Schematic figure of the Play model

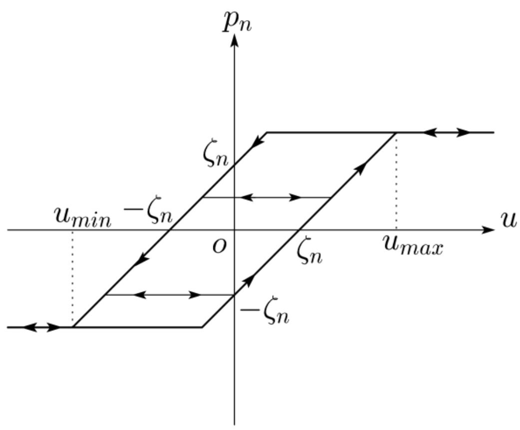

Figure 7 Behaviour of a Play hysteron $\mathrm{p_n}$ with saturation

The Play model described by equations (4) and (5) can be less cumbersome in numerical implementations as compared with the Preisach model. It should be pointed out that Play model is equivalent to the Preisach model in a sense that it has the same capability on describing hysteretic characteristics. In the identification of a Play model, shape function $\mathrm{f_n}$ corresponding to Play hysteron $\mathrm{p_n}$ must also be determined. Let $\Delta\mathrm{p} = (u_{max} - u_{min})/N_p$ be the unit of discretization for outputs of a Play hysteron and let

$$p_{n,m} = m\Delta p + u_{min} + \zeta_n, \qquad (6)$$

be the m-th possible value of $\mathrm{p_n}$. Denoting $\mathrm{f_{n,m}} = \mathrm{f_n}(p_{n,m})$ where $\mathrm{f_n}$ is the n-th shape function corresponding to $\mathrm{p_n}$, the shape function is defined by interpolating the points $(p_{n,m-1}, f_{n,m-1})$ and $(p_{n,m}, f_{n,m})$ using the equation

$$f_n(p) = f_{n,m-1} + \hat{\mu}(n,m)(p - p_{n,m-1}) \qquad (7)$$

in (p, f_n) plane as shown in Fig.8. It can be shown that gradient function $\hat{\mu}(m,n)$ of equation (7) is related to the discretized Preisach distribution function in a manner defined by

$$\hat{\mu}(m,n) = \widehat{K}(n+m-1,m)/\Delta p\,. \qquad (8)$$

The resulting output of the Play model will vary continuously in accordance with the changes of the input signal because the shape function (7) is defined to be a continuous function of p.

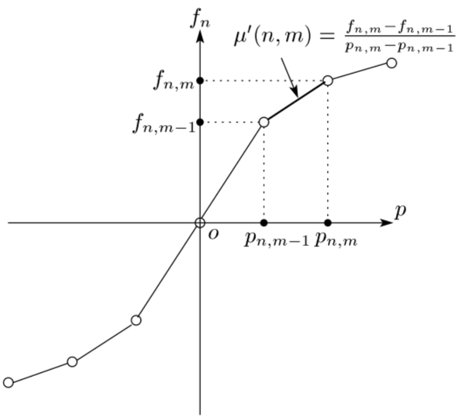

Figure 8 Shape function for a Play hysteron $\mathrm{p_n}$

3.3 Stop model

The Stop model describes the hysteretic relation between the input $u(t)$ and the output $F(u)$ by the equation

$$\mathrm{F(u)} = \sum_{n=1}^{N_s} g_n\left(s_n(u(t))\right), \qquad (9)$$

where $\mathrm{N_s}$ is the number of Stop hysteron operators, $\mathrm{s_n}$ is the n-th Stop hysteron operator, and $\mathrm{g_n}$ is a single-valued shape function for $\mathrm{s_n}$. The Stop hysteron model is shown in Fig.9. The mathematical definition of the Stop hysteron $\mathrm{s_n}$ is described by the equation

$$s_n(u(t)) = \max(\min(u - u^0 + s_n^0, \eta_n), -\eta_n))\,, \qquad (10)$$

where (u_n^0, s_n^0) is the value of (u, s_n) at the previous sampling instance, and η_n is the height of the Stop hysteron s_n. The behavior of s_n with saturation is illustrated in Fig.10. Stop model forms hysteretic loops in clockwise direction whereas Play model forms counter-clockwise loops. Therefore it is a natural consequence to use a Stop model for describing the inverse hysteretic relation between the input and the output.

The determination of a shape function $\mathrm{g_n}(s)$ for a Stop hysteron s_n follows a quite similar procedure as done with the Play model. Let $\Delta\mathrm{s} = (u_{max} - u_{min})/N_s$ be the unit of discretization for the value of a Stop hysteron. As shown in Fig.10, Stop hysteron s_n will vary within the interval $[-\eta_n, \eta_n]$. The m-th possible value of the Stop hysteron $\mathrm{s_n}$ can be expressed by

$$s_{n,m} = m\Delta s - \eta_n \; (m = 1, \cdots, n)\,. \qquad (11)$$

Using the same notation $g_{n,m-1} = g_n(s_{n,m-1})$ as used with the Play model, the shape function $\mathrm{g_n}(s)$ for a Stop hysteron is given by

$$g_n(s) = g_{n,m-1} + \hat{\kappa}(n,m)(s - s_{n,m-1})\,, \qquad (12)$$

where the gradient $\hat{\kappa}$ is defined by

$$\hat{\kappa}(n,m) = \widehat{K}(n-m+1,m)/\Delta s\,. \qquad (13)$$

Equation (12) defines a continuous function over the range where a Stop hysteron varies, which amounts to defining a continuous hysteresis characteristics of a Stop model.

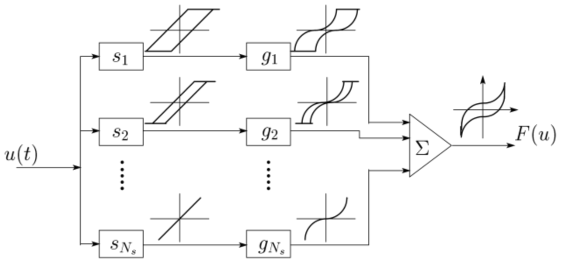

Figure 9 Schematic figure of the Stop model

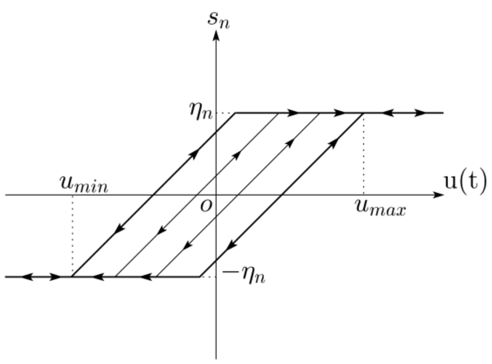

Figure 10 Behaviour of a Stop hysteron with saturation

3.4 Identification of Preisach distribution function

In order to construct a feed-forward controller to suppress the hysteretic behavior, identification of an employed model is necessary to establish a numerical model which reproduces hysteretic behaviour. The identification of a Preisach distribution function will be outlined here since it has been explained above that the Play and the Stop models can be constructed using the Preisach distribution function $\widehat{K}(i,j)$.

Identification of a discretized distribution function is a procedure to retrieve the value of $\widehat{K}(i,j)$ from the measured data. It can be shown that identification of discretized distribution function would result in obtaining the solution of simple linear equations repeatedly by using specifically designed input signal like decaying triangular wave. Details of the procedure is omitted for lack of space. Interested readers are encouraged to see the reference [7].

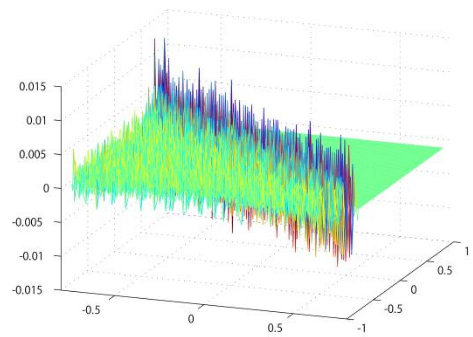

Figure 11 Distribution function for 1Hz input

We have normalized the ranges of input $u(t)$ to the interval[-1,1] and discretized it to 100 portions. Discretization is carried out both for α and β. The Preisach plane should be discretized accordingly resulting in a triangular region with 5,000 cells. Identification of a distribution function for a single input frequency requires the determination of 5,000 values of a distribution function. Fig.11 shows an example of the result of identification whose input signal frequency is 1 [Hz].

4. Inverse hysteresis modeling and the controller design

One approach to compensate the hysteresis of piezo-ceramic actuator is to employ an inverse hysteresis operator as a feed-forward controller of a control system and calculate the appropriate input to cancel the unwanted hysteretic behavior. Resulting feed-forward control system is shown in Fig.12. Inverse hysteresis model can be established using the identified hysteresis model. This section dictates how inverse hysteresis is modeled and used as a feed-forward controller.

Figure 12 Block diagram of a feed-forward control system

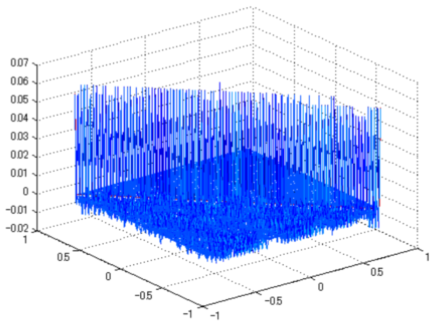

Figure 13 Inverse distribution function for 1Hz input

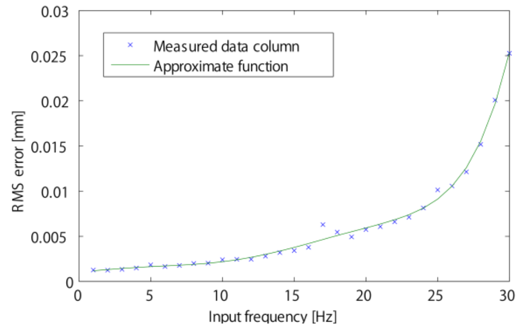

Figure 14 Polynomial approximation of response amplitude

4.1 Construction of inverse distribution function

The simplest way to calculate inverse hysteresis output which is the ideal input to retrieve desired output from the actuator is to use the identified hysteresis model repeatedly to search the input value which corresponds to the desired output. However, evaluating the hysteresis model number of times takes computational time and hence is not suitable for real time control.

One practical idea of hysteresis compensation by feed-forward control in real time is to formulate the model which describes the inverse hysteretic relation. Using the formulation of Preisach model, the problem of establishing inverse hysteresis model turns out to be the identification problem of an inverse distribution function. Identification of an inverse distribution function follows almost the same procedure as explained in section 3.4, except the values of input/output signals used in the calculation are taken from the inverse hysteresis curve which is numerically generated. Fig.13 shows an example of the inverse distribution function for 1[Hz] input identified in a manner as explained above.

4.2 Controller design based on Play model

Recall that the output of a Play model for a particular input sequence can be calculated by using the Preisach distribution function. The Play model based feed-forward controller for hysteresis compensation is itself a Play model whose distribution function is the inverse distribution function $\hat{K}^{-1}(i,j)$ identified in a way as explained in section 4.1.

To reduce the computational load necessary for compensating hysteretic behaviour for a number of different input frequencies, an interpolation method of inverse distribution functions will be proposed. The points plotted using the symbol x in Fig.14 represent the experimental data of RMS tracking error of the piezo-ceramic actuator for sinusoidal inputs whose frequency ranges from 1 to 30 Hz. Note that these data are obtained without control. A 6th order polynomial $\phi(f)$ of a frequency variable f is then fitted to approximate these data.

Let us assume that we have obtained two inverse distribution functions directly for terminal frequencies f_1 and f_2 Hz inputs, denoted by $\hat{K}_{f_1}^{-1}$ and $\hat{K}_{f_2}^{-1}$, respectively. The inverse distribution function for frequency other than f_1 and f_2 can be constructed numerically by interpolating the *a-priori* inverse distribution functions for f_1 and f_2, as schematically depicted in Fig.15. The shape functions of the Play model are accordingly determined by using the interpolated inverse distribution function. The Play model so constructed will be referred to as the enhanced Play model of the inverse hysteresis.

4.3 Controller design based on Stop model

It can be said that Stop model is well-suited for describing an inverse hysteresis loop because Stop hysterons move in clockwise direction, while Play hysterons move in an opposite direction. However, using the normal Preisach distribution function with a Stop model does not work as desired because Stop model thus configured will not exhibit the inverse hysteretic relation but will form a clockwise loop whose shape is identical to the normal hysteretic relation. It is hence also necessary to identify the inverse distribution function to establish the Stop model based feed-forward controller.

The enhanced Stop model capable of compensating rate dependent inverse hysteresis when used as a feed-forward controller is a Stop model whose distribution function for unidentified frequency *f* Hz is calculated by interpolating the two known inverse distribution functions of input frequency f_1Hz and f_2Hz.

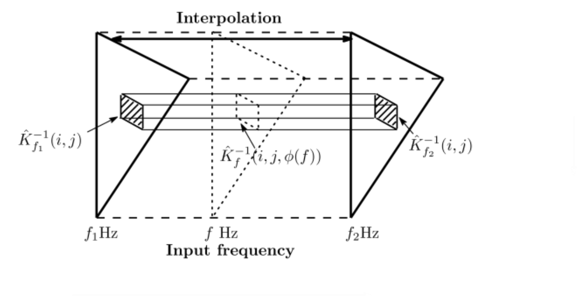

Figure 15 Interpolation of inverse distribution functions

5. Experimental result

With the feed-forward controllers based on the enhanced Play and Stop models developed above, experimental results of responses of the piezo-ceramic actuator for sinusoidal reference inputs and exponentially decaying sinusoidal reference inputs are summarized in Figs. 16 and 17, respectively. RMS error is used as a performance index in each figure. The experiment with exponentially decaying reference inputs is conducted to check the control performance of the proposed controller for minor loop hysteresis, for exponentially decaying inputs would be the cause of minor hysteresis loops without control.

For compensation of major loop hysteresis, both the Play and the Stop model based controllers show good performance for input frequency lower than 25Hz. They can maintain RMS error below 0.5 μm as Fig.16 shows. However, the performance of proposed controllers degrade slightly at 17Hz and significantly at/around 27Hz, resulting in relatively large RMS error.

For compensation of minor loop hysteresis, similar tendency can be observed from the results shown in Fig.17. Both the Play and the Stop model based controllers show good performance for input frequency lower than 25Hz. They can maintain RMS error below 0.5 μm. However, as can also be observed in the major loop experiment, performance degradation again occurs for input frequencies 17Hz and around 27Hz. It should be noted that the Stop model based controller out-performs the Play model based controller by 50% for high input frequencies, which is a situation observed only in the minor loop experiment. It is highly presumable that performance degradation for some specific input frequency might be caused by the resonance of piezo-ceramic actuator.

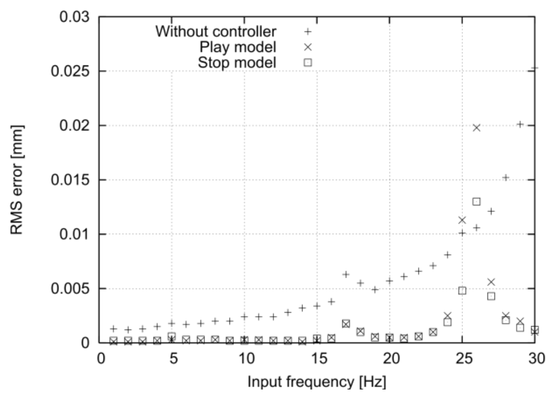

Figure 16 Compensation performance for major loop hysteresis

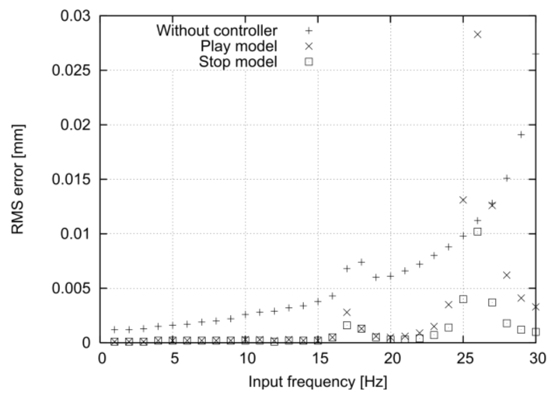

Figure 17 Compensation performance for minor loop hysteresis

6. Conclusion

The inverse hysteresis modeling for rate-dependent hysteresis and its application to tracking control of a piezo-ceramic actuator have been presented in the paper. The interpolation method for constructing inverse hysteresis mapping which is valid for various frequencies without experimental data has been proposed and the inverse model thus constructed has been employed in a hysteresis compensation control system as a feed-forward controller. The results of the experiment obtained by using the proposed controllers indicate that the proposed controllers work well for almost all different input frequencies tested. The proposed method is highly effective for reducing the amount of off-line computation and requires less time in designing the controller.

However, it is also true that performance degradation has been observed for some specific input frequency. The authors presume that it has something to do with the resonance frequency of piezo actuator. Our future work will include the improvement of this performance degradation by modeling the resonant characteristics of piezo-actuators in an appropriate way and then proceed to the design of controller improving the tracking performance in these frequencies.

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